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Braneworlds, conformal fields and the gravitons

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Abstract
We investigate the dynamics of Randall–Sundrum AdS_5 braneworlds with five-dimensional conformal matter fields. In the scenario with a compact fifth dimension the class of conformal fields with weight -4 is associated with exact five-dimensional warped geometries which are stable under radion field perturbations and describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic dark energy. We analyse the graviton mode fluctuations around this class of background solutions and determine their mass eigenvalues and wavefunctions from a Sturm–Liouville problem. We show that the localization of gravity is not sharp enough for large mass hierarchies to be generated. We also discuss the physical bounds imposed by experiments in particle physics, in astrophysics and in precise measurements of the low energy gravitational interaction.

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1. Introduction

In the AdS_5 Randall–Sundrum (RS) scenario [1, 2] our visible four-dimensional (4D) Universe is a 3-brane world embedded in a Z_2 symmetric five-dimensional (5D) anti-de Sitter (AdS) space. In the RS1 model [1] the fifth dimension is compact and there are two 3-brane boundaries. In this setting gravity is exponentially localized near the hidden positive tension brane and decays towards the observable negative tension brane. The hierarchy problem is then reformulated as an exponential hierarchy between the weak and Planck scales [1]. In the RS2 model [2] the AdS_5 orbifold has an infinite fifth dimension and a single visible positive tension brane to which the gravitational field is exponentially bound.

In the visible brane the low energy theory of gravity is 4D general relativity and the cosmology may be Friedmann–Robertson–Walker [1–10]. In the RS1 model this requires the stabilization of the radion mode with, for example, a 5D scalar field [3, 6, 9, 10]. Using the Gauss–Codazzi formulation [11, 12] many other braneworld solutions have been discovered although a number of them have not yet been associated with exact 5D spacetimes [13–16].

In this paper we continue the research about the dynamics of a spherically symmetric RS 3-brane when conformal matter fields propagate in the bulk [17–20] (see also [21]). Some time ago [17, 18] we discovered a new class of exact 5D dynamical warped solutions which is associated with conformal fields characterized by an energy–momentum tensor of weight −4. These solutions were shown to describe on the brane the dynamics of inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter [17, 18]. The latter in particular refers to a perfect dark energy fluid and describes the accelerated expansion of our Universe. The radion may be stabilized by a sector of the conformal bulk fields of weight −4 while another sector generates the dynamics on the brane. The stabilization requires a bulk fluid sector with a constant negative 5D pressure and involves new warp functions [20]. If the theory of gravity on the brane deviates from that of Einstein the existence of such dynamical geometries requires the presence of non-conformal matter fields confined to the brane [22]. In this work we analyse the graviton field perturbations around this class of background geometries and determine their mass eigenvalues and wavefunctions from a Sturm–Liouville problem. We show that gravity is not sufficiently localized near the positive tension branes to be able to generate large mass hierarchies. We also discuss the physical bounds imposed by experiments in particle physics, in astrophysics and in precise measurements of Newton's law of gravity.

2. 5D Einstein equations and conformal fields

The most general non-factorizable dynamical metric consistent with the Z_2 symmetry in the fifth dimension and with 4D spherical symmetry on the brane is given by

$$d\tilde{s}_5^2 = \Omega^2 \left(dz^2 - e^{2A}\, dt^2 + e^{2B}\, dr^2 + R^2\, d\Omega_2^2\right), \tag{1}$$

where (t, r, θ, ϕ, z) are the coordinates mapping the AdS_5 orbifold. In this set z is related to the Cartesian coordinate y by $z = l\, e^{y/l}$, $y > 0$, with l being the AdS radius. The functions $\Omega = \Omega(t, r, z)$, $A = A(t, r, z)$, $B = B(t, r, z)$ and $R = R(t, r, z)$ are Z_2 symmetric. $R(t, r, z)$ represents the physical radius of the 2-spheres and Ω is the warp factor characterizing a global conformal transformation on the metric.

In the RS1 model the classical dynamics is defined by the 5D Einstein equations,

$$\tilde{G}_\mu^\nu = -\kappa_5^2 \left\{ \Lambda_B \delta_\mu^\nu + \frac{1}{\sqrt{\tilde{g}_{55}}}[\lambda\delta\,(z - z_0) + \lambda'\delta(z - z'_0)]\left(\delta_\mu^\nu - \delta_5^\nu\delta_\mu^5\right) - \tilde{\mathcal{T}}_\mu^\nu \right\}, \tag{2}$$

where Λ_B is the negative bulk cosmological constant and $\kappa_5^2 = 8\pi / M_5^3$ with M_5 being the fundamental 5D mass scale. λ, λ' are the brane tensions and $\tilde{\mathcal{T}}_\mu^\nu$ is the stress–energy tensor of the matter fields which in 5D is conserved:

$$\tilde{\nabla}_\mu \tilde{\mathcal{T}}_\nu^\mu = 0. \tag{3}$$

For a general 5D metric $\tilde{g}_{\mu\nu}$ (2) and (3) form a complex system of differential equations. To solve it let us introduce some simplifying assumptions [20]. First consider that the bulk matter is described by conformal fields with weight s. Under the conformal transformation $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ the stress–energy tensor satisfies $\tilde{\mathcal{T}}_\mu^\nu = \Omega^{s+2}\mathcal{T}_\mu^\nu$. Then separate the conformal tensor $\tilde{\mathcal{T}}_\mu^\nu$ in two sectors $\tilde{T}_\mu^\nu$ and $\tilde{U}_\mu^\nu$ with the same weight s, $\tilde{\mathcal{T}}_\mu^\nu = \tilde{T}_\mu^\nu + \tilde{U}_\mu^\nu$, where $\tilde{T}_\mu^\nu = \Omega^{s+2} T_\mu^\nu$ and $\tilde{U}_\mu^\nu = \Omega^{s+2} U_\mu^\nu$, and take $s = -4$. Finally, consider $A = A(t, r)$, $B = B(t, r)$, $R = R(t, r)$ and $\Omega = \Omega(z)$. Then (2) leads to [20]

$$G_a^b = \kappa_5^2 T_a^b, \tag{4}$$

$$G_5^5 = \kappa_5^2 T_5^5, \tag{5}$$

$$6\Omega^{-2}(\partial_z\Omega)^2 + \kappa_5^2\Omega^2\Lambda_{\mathrm{B}} = \kappa_5^2 U_5^5, \tag{6}$$

$$\left\{3\Omega^{-1}\partial_z^2\Omega + \kappa_5^2\Omega^2\{\Lambda_{\mathrm{B}} + \Omega^{-1}[\lambda\delta(z-z_0) + \lambda'\delta(z-z'_0)]\}\right\}\delta_a^b = \kappa_5^2 U_a^b, \tag{7}$$

where the Latin indices represent the 4D coordinates t, r, θ and ϕ. On the other hand, from (3) we also obtain [20]

$$\nabla_a T_b^a = 0 \tag{8}$$

and the equations of state $2T_5^5 = T_a^a$, $2U_5^5 = U_a^a$. U_μ^ν turns out to be a constant diagonal tensor, $U_\mu^\nu = \mathrm{diag}(-\bar{\rho}, \bar{p}_{\mathrm{r}}, \bar{p}_{\mathrm{T}}, \bar{p}_{\mathrm{T}}, \bar{p}_5)$, with the density $\bar{\rho}$ and pressures $\bar{p}_{\mathrm{r}}, \bar{p}_{\mathrm{T}}, \bar{p}_5$ satisfying $\bar{p}_5 = -2\bar{\rho} = 2\bar{p}_{\mathrm{r}} = 2\bar{p}_{\mathrm{T}}$. Note that $\nabla_a U_b^a = 0$ is an identity. If $T_\mu^\nu = \mathrm{diag}(-\rho, p_{\mathrm{r}}, p_{\mathrm{T}}, p_{\mathrm{T}}, p_5)$, where ρ, p_{r}, p_{T} and p_5 are, respectively, the density and pressures then its equation of state is re-written as

$$\rho - p_{\mathrm{r}} - 2p_{\mathrm{T}} + 2p_5 = 0. \tag{9}$$

3. Exact 5D warped solutions

The AdS$_5$ braneworld dynamics is defined by the solutions of (4) to (9) [20]. Solving (6) and (7) we obtain

$$\Omega(y) = \mathrm{e}^{-|y|/l}\left(1 + p_{\mathrm{B}}^5\,\mathrm{e}^{2|y|/l}\right), \qquad \lambda = \lambda_{\mathrm{RS}}\frac{1-p_{\mathrm{B}}^5}{1+p_{\mathrm{B}}^5}, \qquad \lambda' = -\lambda_{\mathrm{RS}}\frac{1-p_{\mathrm{B}}^5\exp(2\pi r_{\mathrm{c}}/l)}{1+p_{\mathrm{B}}^5\exp(2\pi r_{\mathrm{c}}/l)}, \tag{10}$$

where $p_{\mathrm{B}}^5 = \bar{p}_5/(4\Lambda_{\mathrm{B}})$, $\lambda_{\mathrm{RS}} = 6\big/\left(l\kappa_5^2\right)$ and r_{c} is the RS compactification scale.

To determine the dynamics on the brane we solve (4) and (5) when T_μ^ν satisfies (8) and (9) [20]. Note that as long as p_5 balances ρ, p_{r} and p_{T} according to (5) and (9), the 4D equation of state is not constrained. Three examples corresponding to inhomogeneous dust, generalized dark radiation and homogeneous polytropic matter were considered in [17, 18]. The latter describes the dynamics on the brane of dark energy in the form of a polytropic fluid. The 5D dark energy polytropic solutions are [18]

$$\mathrm{d}\tilde{s}_5^2 = \Omega^2\left[-\mathrm{d}t^2 + S^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega_2^2\right)\right] + \mathrm{d}y^2, \tag{11}$$

where S satisfies $S\dot{S}^2 = \kappa_5^2(\eta S^{3-3\alpha} + a)^{\frac{1}{1-\alpha}}/3 - kS$. α and η characterize different polytropic phases. For $-1 \leqslant \alpha < 0$, the fluid is in its generalized Chaplygin phase.

4. Radion stability

Provided the equation of state of the conformal fluid is independent of the radion perturbation, these solutions are stable for a range of the model parameters if $p_{\mathrm{B}}^5 > 0$ [20]. Using $x = y/r_c$ the radion mass is given in terms of the dimensionless radion mass parameter M (see figure 1):

$$m_\gamma = \frac{1}{r_c}\sqrt{\frac{4|M|}{3\int_{-\pi}^{\pi}\mathrm{d}x\,\Omega^2}}, \qquad M = \lambda r_{\mathrm{c}}\kappa_5^2\Omega^4(0) + \lambda' r_{\mathrm{c}}\kappa_5^2\Omega^4(\pi) - \frac{6r_{\mathrm{c}}^2}{l^2}\int_{-\pi}^{\pi}\mathrm{d}x\,\Omega^4. \tag{12}$$

For $p_{\mathrm{B}}^5 > 0$, the stability of the AdS$_5$ braneworlds also depends on the dimensionless ratio l/r_{c}. For $l/r_{\mathrm{c}} < 1.589\ldots$ all solutions turn out to be unstable. Stable universes begin to

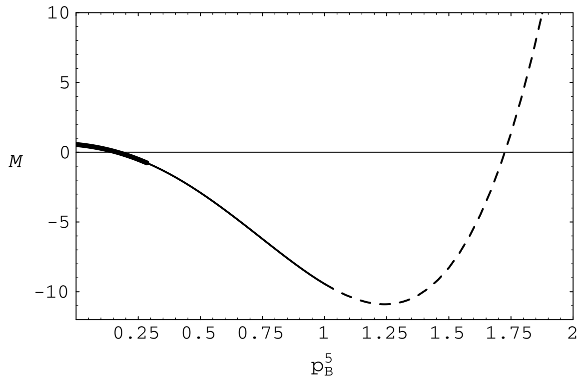

Figure 1. Plot of the radion mass parameter M for $l/r_c = 5$. Thick line—$0 < p_B^5 \leqslant e^{-2\pi/5} : \lambda > 0, \lambda' \leqslant 0$; thin line—$e^{-2\pi/5} < p_B^5 \leqslant 1 : \lambda \geqslant 0, \lambda' > 0$; dashed line—$p_B^5 > 1 : \lambda < 0, \lambda' > 0$.

appear at $l/r_c = 1.589, \ldots, p_B^5 = 0.138\ldots$. For $l/r_c > 1.589, \ldots$, we find stable solutions for an interval of p_B^5 (see in figure 1 the example of $l/r_c = 5$) which increases with l/r_c. For large enough but finite l/r_c, the stability interval approaches the limit $]0.267, \ldots, 3.731, \ldots[$. Naturally, $M \to 0$ if $l/r_c \to \infty$.

The particle physics bound for the radion mass is $m_\gamma > 35$–120 GeV [23]. For fixed l/r_c, the order of magnitude bound $m_\gamma > 100$ GeV requires a small fifth dimension, $r_c < 1 \times 10^{-15}$ mm. Since the 5D mass scale is related to the 4D Planck mass $M_P^2 = 8\pi\big/\kappa_4^2 \sim 1 \times 10^{19}$ GeV by $M_5^3 r_c \int_{-\pi}^{\pi} dx\, \Omega^2 = M_P^2$, we conclude that $M_5 > 2 \times 10^{10}$ TeV. Consequently this radion is invisible in astrophysical processes which require at least $m_\gamma > 100$ MeV [23]. It is also invisible in precision Newton's law measurements which imply $m_\gamma > 6.25 \times 10^{-4}$ eV [23].

If the scale M_5 is high enough then the radion may be light. The direct bound from particle physics is $M_5 > 1$–2 TeV [23]. From astrophysics it may be as high as $M_5 > 1$–200 TeV [23]. Now $m_\gamma > 6.25 \times 10^{-4}$ eV corresponds to a large fifth dimension with $r_c < 160\ \mu$m. Then the 5D scale is indeed high, $M_5 > 4 \times 10^5$ TeV.

These results are a consequence of the absence of a large mass hierarchy, $m = \Omega(\pi r_c) m_0$, $\Omega(\pi r_c) \sim 1 - 100$, and as order of magnitude estimates they hold for all the p_B^5 stability range.

5. The gravitons

To analyse the graviton field perturbations around the background $\bar{g}_{\mu\nu}$ let us write the classical 5D Einstein equations as

$$\tilde{G}_{\mu\nu} = -\kappa_5^2 \tilde{\mathcal{M}}_{\mu\nu}, \tag{13}$$

where the energy–momentum tensor $\tilde{\mathcal{M}}_{\mu\nu}$ is

$$\tilde{\mathcal{M}}_{\mu\nu} = \frac{1}{\sqrt{\tilde{g}_{55}}}[\lambda\delta(y - y_0) + \lambda'\delta(y - y'_0)]\left(\tilde{g}_{\mu\nu} - \delta_\mu^5 \tilde{g}_{5\nu} - \tilde{g}_{\mu 5}\delta_\nu^5 + \tilde{g}_{55}\delta_\mu^5\delta_\nu^5\right) + \Lambda_B \tilde{g}_{\mu\nu} - \tilde{\mathcal{T}}_{\mu\nu}. \tag{14}$$

Consider the graviton perturbation $h_{\mu\nu}$ such that $\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. Using $\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h$ where $h = h^{\mu}_{\mu}$ we find

$$\tilde{G}_{\mu\nu} = \bar{G}_{\mu\nu} - \tfrac{1}{2}(\bar{\nabla}^2\hat{h}_{\mu\nu} + \bar{g}_{\mu\nu}\bar{\nabla}^{\alpha}\bar{\nabla}^{\beta}\hat{h}_{\alpha\beta}) + \bar{\nabla}_{(\mu}\bar{\nabla}^{\alpha}\hat{h}_{\alpha\nu)} + \bar{R}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} + \bar{G}^{\beta}_{(\mu}\hat{h}_{\nu)\beta} + \tfrac{1}{6}\bar{R}\hat{h}\bar{g}_{\mu\nu}. \quad (15)$$

On the other hand $\tilde{\mathcal{M}}_{\mu\nu} = \bar{\mathcal{M}}_{\mu\nu} + \delta\mathcal{M}_{\mu\nu}$. Since $\bar{G}_{\mu\nu} = -\kappa_5^2\bar{\mathcal{M}}_{\mu\nu}$ we obtain

$$-\tfrac{1}{2}(\bar{\nabla}^2\hat{h}_{\mu\nu} + \bar{g}_{\mu\nu}\bar{\nabla}^{\alpha}\bar{\nabla}^{\beta}\hat{h}_{\alpha\beta}) + \bar{\nabla}_{(\mu}\bar{\nabla}^{\alpha}\hat{h}_{\alpha\nu)} + \bar{R}^{\alpha\beta}_{\mu\nu}\hat{h}_{\alpha\beta} = -\bar{G}^{\beta}_{(\mu}\hat{h}_{\nu)\beta} - \tfrac{1}{6}\bar{R}\hat{h}\bar{g}_{\mu\nu} - \kappa_5^2\delta\mathcal{M}_{\mu\nu}. \quad (16)$$

Assume a flat brane background $\bar{g}_{ab} = \Omega^2(y)\eta_{ab}$, $\bar{g}_{55} = 1$, $\bar{g}_{5a} = \bar{g}_{a5} = 0, \forall a$. Consider also the graviton perturbation $\tilde{g}_{ab} = \bar{g}_{ab} + h_{ab}$, $\tilde{g}_{55} = \bar{g}_{55}$, $\tilde{g}_{5a} = \bar{g}_{5a}$ and the RS gauge $\partial^a h_{ab} = h = 0$. Then

$$-\tfrac{1}{2}\bar{\nabla}^2 h_{\mu\nu} + \bar{R}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \bar{G}^{\beta}_{(\mu}h_{\nu)\beta} = -\kappa_5^2\delta\mathcal{M}_{\mu\nu}. \quad (17)$$

Expanding, we find

$$\left[-\frac{1}{2}\left(\Omega^{-2}\Box_4 + \partial_y^2\right) + 4\frac{\partial_y^2\Omega}{\Omega} + 4\left(\frac{\partial_y\Omega}{\Omega}\right)^2\right]h_{ab} = -\kappa_5^2\delta\mathcal{M}_{ab}. \quad (18)$$

The conformal 5D energy–momentum tensor $\tilde{\mathcal{T}}_{\mu\nu}$ is

$$\tilde{\mathcal{T}}_{ab} = \frac{2\Lambda_{\rm B}p_{\rm B}^5}{\Omega^2}\tilde{g}_{ab}, \qquad \tilde{\mathcal{T}}_{a5} = \tilde{\mathcal{T}}_{5a} = 0, \qquad \tilde{\mathcal{T}}_{55} = \frac{4\Lambda_{\rm B}p_{\rm B}^5}{\Omega^2}\tilde{g}_{55}. \quad (19)$$

Then the energy–momentum tensor perturbation is

$$\delta\mathcal{M}_{ab} = \Lambda_{\rm B}\left(1 - \frac{2p_{\rm B}^5}{\Omega^2}\right)h_{ab} + [\lambda\delta(y - y_0) + \lambda'\delta(y - y_0')]h_{ab}. \quad (20)$$

Using the warp equations (6), (7) in the y coordinate and writing the graviton wavefunction as $h_{ab} = \mathrm{e}^{\mathrm{i}p\cdot x}\psi_{ab}(y)$, where $p^2 = -m^2$, we obtain the following Sturm–Liouville problem:

$$-\frac{1}{2}\partial_y^2\psi_{ab} + \frac{2}{l^2}\left[1 - 2p_{\rm B}^5\,\mathrm{e}^{2|y|/l}\left(1 + p_{\rm B}^5\,\mathrm{e}^{2|y|/l}\right)^{-2}\right]\psi_{ab} - \frac{2}{l}\frac{1 - p_{\rm B}^5}{1 + p_{\rm B}^5}\delta(y)\psi_{ab}$$
$$- \frac{2}{l}\frac{1 - p_{\rm B}^5\,\mathrm{e}^{2\pi r_c/l}}{1 + p_{\rm B}^5\,\mathrm{e}^{2\pi r_c/l}}\delta(y - \pi r_c)\psi_{ab} = m^2\,\mathrm{e}^{2|y|/l}\left(1 + p_{\rm B}^5\,\mathrm{e}^{2|y|/l}\right)^{-2}\psi_{ab}. \quad (21)$$

The eigenvalues m^2 are positive or zero leading to $m \geqslant 0$. The set is infinite but discrete, $m_i, i = 0, 1, \ldots + \infty$. There is just one massless graviton, $m_0 = 0$. It has a positive wavefunction ψ_0 in the fifth dimension. This wavefunction is localized near the positive tension branes. The massive gravitons have oscillating wavefunctions $\psi_i, i = 1, \ldots + \infty$. Their masses and mass splittings decrease when r_c increases. For the solutions with a stabilized radion there is no hierarchy between graviton wavefunctions (see figure 2) so that gravity is not strongly localized.

Like the radion the massive gravitons may be above the TeV scale. Their mass splittings have the same order of magnitude. The radion and the less massive graviton have similar masses. The massive gravitons may also be light. Newton's potential is

$$V(r) = -\frac{G_4 m m'}{r}\left(1 + \frac{|\psi_1|^2}{|\psi_0|^2}\,\mathrm{e}^{-m_1 r} + \cdots\right). \quad (22)$$

Since $|\psi_1|^2/|\psi_0|^2 \sim 1$ we must have $r_c < 160\ \mu$m and $m_1 > 1 \times 10^{-3}$ eV.

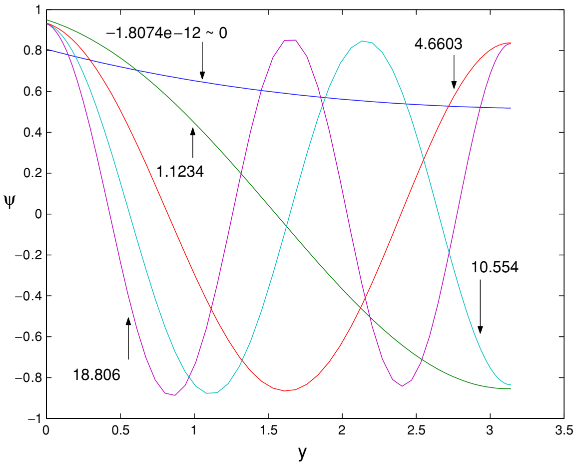

Figure 2. Plot of graviton wavefunctions ψ for $l/r_c = 5$ and $p_B^5 = 0.25$. Shown are the massless graviton and the first four massive gravitons. The mass scale is set by $r_c = 100\ \mu\text{m} = 1 \times 10^3\ \text{eV}^{-1}$. (This figure is in colour only in the electronic version)

6. Conclusions

In this paper we have analysed exact 5D solutions describing the dynamics of AdS_5 braneworlds when conformal fields of weight −4 exist in the bulk. We have considered solutions for which gravity is localized near the brane and the dynamics on the brane is, for example, that of inhomogeneous dust, generalized dark radiation and homogeneous polytropic dark energy. We have seen that the radion may be stabilized using only the conformal bulk fields of weight −4 which generate the dynamics on the brane. This requires invariance of their equation of state under the radion perturbation, a stabilizing sector with a constant negative 5D pressure and new warp functions. We have also discussed graviton perturbations and determined their mass eigenvalues and wavefunctions from a Sturm–Liouville problem. Besides a massless graviton localized on the positive tension branes we have seen that this scenario involves an infinite discrete set of increasingly massive gravitons. We have shown that the new stabilizing warp functions are unable to generate a large mass hierarchy and that gravity is not strongly localized near the branes. Possibilities of overcoming this problem are the introduction of supersymmetry [24] or of a non-conformal 5D scalar field [3] to stabilize the radion field. This analysis is left for future research. Finally we have also shown that to satisfy the current observational constraints [23] the radion and the less massive graviton may either be heavier than $\sim$100 GeV corresponding to a small fifth dimension $\sim 1 \times 10^{-15}$ mm or light with mass above $\sim 1 \times 10^{-3}$ eV corresponding to a compactification scale of the order of 100 μm.

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